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# Blobs strengthen repetition but weaken symmetry

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## Abstract

The human visual system is more sensitive to symmetry than to repetition. According to the so-called holographic approach [J. Math. Psychol. 35 (1991) 151; Psychol. Rev. 103 (1996) 429; Psychol. Rev. 106 (1999) 622], however, this perceptual difference between symmetry and repetition depends strongly on spatial scaling. This was tested in three experiments, using symmetry and repetition stimuli that consisted of black and white patches, with patch size as the critical variable. In Experiment 1, patch size was increased in the entire pattern, yielding fewer but larger patches (or *blobs*). This is known to have hardly any effect on symmetry but, as found now, it does have a strengthening effect on repetition but, as double-checked in experiment 3, it can weaken symmetry. These results agree with the holographic approach, and enable an integration of computational, algorithmic, and implementational aspects of vision.

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## 1. Introduction

Visual regularities such as symmetry and repetition are important cues in the perceptual structuring of the visible world (cf. Attneave, 1954; Garner, 1974; Klix, 1971; Koffka, 1962; Leeuwenberg, 1971; Palmer, 1983; Wagemans, 1995; Wertheimer, 1923). That is, detection of these regularities is an integral part of the general perceptual interpretation process that is applied to any visual input. This is not to say, however, that all visual regularities are detected with equal ease. For instance, human observers are much more sensitive to symmetry than to repetition (Bruce & Morgan, 1975; Corballis & Roldan, 1974; Fitts, Weinstein, Rappaport, Anderson, & Leonard, 1956; Julesz, 1971; Zimmer, 1984). In this study, we go into more detail on this well-known phenomenon, to get more insight in the mechanisms underlying regularity detection. The theoretical and empirical accounts will be reviewed using Marr's (1982) distinction between the computational level of description (specifying a system's goal), the algorithmic level of description (specifying a system's method), and the implementational level of description (specifying a system's means).

We report three experiments, in which we tested the effect of spatial scaling on the detectability of symmetry and repetition. In all three experiments, the stimuli consisted of black and white patches. In the first experiment, we increased the size of all patches in a stimulus, yielding fewer but larger patches (or blobs). In the second and third experiments, we increased the size of the patches in subpatterns only, yielding salient blob areas.

The symmetry manipulation in the first experiment is known in the literature (see next section) but, to our knowledge, the other symmetry and repetition manipulations are not. That is, these other manipulations yield novel stimuli, that we expected to reveal thus far unknown differential effects for symmetry and repetition.

This expectation was triggered by the so-called holographic approach. This approach is based on a new formalization of visual regularity (for details, see van der Helm & Leeuwenberg, 1991), and comprises two coherent regularity-detection models at the computational and algorithmic levels of description, respectively (for details, see van der Helm & Leeuwenberg, 1996, 1999). The algorithmic model is sketched later on, in the introduction to the second experiment. At this moment,

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it may suffice to mention that it is a faithful algorithmic translation of the computational model that is sketched in the now following introduction to the first experiment.

# 2. Number effects

van der Helm and Leeuwenberg's (1996) computational model reflects the idea that the detectability of a visual regularity in a stimulus is determined by the representational strength (or *weight of evidence*; see McKay, 1969) of this regularity. More specifically, the model quantifies the strength of a regularity by W = E/n, in which *n* is the number of stimulus elements, while E is the number of the so-called holographic identities that, according to van der Helm and Leeuwenberg's (1991) formalization, constitute the regularity. This implies, for symmetry, that E is the number of element pairs that form a symmetry pair, whereas, for repetition, E is the number of repeats minus one. Thus, for a perfect symmetry on *n* elements, E = n/2 and W = 0.5. That is, W is independent of the number of stimulus elements. For a perfect *m*-fold repetition (i.e., *m* repeats) on *n* elements, however, E = m - 1 and W =(m-1)/n. That is, for fixed m, W depends strongly on the number of stimulus elements. In this article, we consider twofold repetition only, for which W = 1/n.

Hence, by way of this computational model, the holographic approach predicts the well-known phenomenon that symmetry is better detectable than repetition: Generally, symmetry has a higher W value than repetition. In addition, the holographic approach predicts that this difference between symmetry and repetition depends on the number of stimulus elements. That is, it predicts a number effect in repetition but not symmetry. Both predictions contrast with, for instance, the predictions by the so-called transformational approach (Palmer, 1983) and by the so-called bootstrap model (Wagemans, van Gool, Swinnen, & van Horebeek, 1993), which present alternative models at the computational level and the algorithmic level, respectively. The internal structures of symmetry and repetition, as postulated in the transformational approach and in the bootstrap model (see Fig. 1), simply do not allow such differentiations between symmetry and repetition.

It is true that, at the implementational level, similar differentiations might be attributable to, in particular, the factor called proximity or local attention. However, it remains to be seen whether this factor really provides an alternative explanation, or just a compatible explanation at an alternative level of description (see Section 7). Furthermore, the holographic model has considerable explanatory power in a much broader domain that includes not only perfect regularities, but also perturbed regularities and combinations of regularities (see van der Helm & Leeuwenberg, 1996). Currently relevant is that other studies already investigated number effects for symmetry, but not yet for repetition. This is discussed next.

The empirical literature consistently shows that there is indeed hardly a number effect in symmetry. That is, minor or no effects were found when varying the number of line segments in contour shapes (Baylis & Driver, 1994), or when varying the number of dots in dot patterns (Wenderoth, 1996), or when scaling up patch size in patch patterns, yielding fewer but larger patches (Dakin & Watt, 1994; Oomes, 1998; Tapiovaara, 1990). The latter manipulation is the one we used in the experiment reported here, so we expect to find the same



Fig. 1. Internal structure of symmetry and repetition according to various approaches. Every rectangle and square indicates a substructure. (A) The transformational approach. For symmetry, the correspondence between two pattern halves is captured by a 180° 3D rotation about the symmetry axis, and in repetition, it is captured by a translation. This implies that both symmetry and repetition get a block structure, in which each pattern half constitutes one substructure. (B) The bootstrap model. A pair of corresponding points in symmetry or repetition forms a virtual line, and two such virtual lines form a virtual trapezoid in symmetry, whereas they form a virtual parallelogram in repetition. These correlation quadrangles form the anchors for the detection process and imply that both symmetry and repetition get a point structure, in which each element constitutes one substructure. (C) The holographic approach. Representationally, symmetry is constituted by the relationships between corresponding points, and repetition is constituted by the relationships between the repeats. This implies that mirror symmetry gets a point structure, but repetition a block structure.



Fig. 2. Examples of different types of repetition. (A) A single surface pattern in which two contour parts are identical under translation with reversed colouring at either sides of these contour parts; this type is considered by Baylis and Driver (1994). (B) A pattern consisting of two identical juxtaposed subpatterns; this type is considered in the experiments reported here.

result, which would be in line with the holographic approach.

Regarding a number effect in repetition, we know of no empirical studies in the literature. It is true that Baylis and Driver (1994) concluded to a strong number effect in repetition, but their stimuli constitute what we would call anti-repetition. They considered single-surface patterns in which two contour parts were identical under translation but with reversed coloring at either sides of these contour parts. In the first experiment, however, we considered patterns consisting of two identical juxtaposed subpatterns (see Fig. 2). The latter patterns exhibit the conventional type of repetition, for which the holographic approach predicts a number effect. The main objective of the first experiment is precisely to test this holographic prediction.

## 3. Experiment 1

# 3.1. Method

#### 3.1.1. Subjects

Twelve naive observers (five males and seven females) participated in the experiment. All the subjects were undergraduate or postgraduate students at the University of Nijmegen. They were either paid or received course credits. They were aged between 18 and 31 and had normal or corrected-to-normal acuity.

# 3.1.2. Stimuli

Three types of stimulus condition were produced: symmetry, repetition and random. The test stimuli were black and white Gaussian blob patterns of  $500 \times 510$ pixels. The luminance of the black patches was  $1.4 \text{ cd/m}^2$ and the luminance of the white patches was  $86 \text{ cd/m}^2$ . Each stimulus subtended  $5.71^\circ \times 5.83^\circ$  of the visual field. For the random condition, the coarseness of the Gaussian blobs in the patterns was varied as follows: An image was filled with random Gaussian noise and blurred with Gaussian filters with eight different radii (2, 4, 6, 8, 10, 12, 14, 16 pixels). Finally, the pattern was thresholded to get black and white images. For the symmetry and repetition conditions, the right-hand halves of the random patterns were replaced by mirrored respectively identical versions of the left-hand halves.

In the random patterns, the Gaussian blobs exhibit contour continuity across the vertical midline. This is also the case in the symmetry patterns, but not in the repetition patterns. Therefore, in the repetition detection task the right-hand half of the random patterns was 180° rotated, so that blob-contour continuity could not give a cue.

In all conditions, the two halves of each stimulus were separated by a 10-pixel-wide vertical separation bar. Without this separation bar, detection of the repetition patterns would have been extremely difficult, especially at finer scales. Using this separation bar, symmetry and repetition detection became more comparable. That is, spatial separation between the two halves of a stimulus makes symmetry detection more difficult, but repetition detection easier (Corballis & Roldan, 1974). Fig. 3 shows some examples of experimental stimuli in each condition.

Five different stimuli were produced for each scale in all three conditions. Each stimulus was presented three times at two stimulus presentation times (50 and 80 ms). Thus, one experimental block included 480 stimuli: [two conditions (symmetry and random or repetition and random)  $\times$  8 scales  $\times$  2 presentation times  $\times$  5 patterns]  $\times$  3.

#### 3.1.3. Apparatus

A standard PC with a Philips 109B monitor using a  $1024 \times 1280$  pixels resolution presented the stimuli. The stimuli were displayed on a gray background with 70 cd/m<sup>2</sup> luminance. The subjects viewed the screen at a



Fig. 3. Three example stimuli from each condition in Experiment 1: (A) symmetrical stimuli and (B) repetition stimuli.

distance of 114 cm and a button-box was used to record their responses.

## 3.1.4. Procedure

The experiment consisted of two blocks: symmetry and repetition. The order of type of blocks was counterbalanced over subjects. The subjects were instructed to discriminate symmetry or repetition from random stimuli by pressing the appropriate button on the button box. It was emphasized to the subjects that fixation should be maintained throughout each trial and responses should be made as quickly and accurately as possible.

Before each block, the subjects were given a number of practicing trials with feedback about the correctness of their response. During the experiment, subjects no longer got feedback. Before the stimulus appeared, a fixation cross was presented centered on the screen. Two stimulus presentation times were used, namely, 50 and 80 ms. A mask (grid) was briefly (10 ms) presented after each stimulus to obliterate afterimages.

# 3.2. Results

We measured reaction time (RT), and we computed d'as measure of discriminability (Swets, 1964). The threefactorial design had, as independent variables, presentation time (two levels: 50 and 80 ms), regularity (two levels: repetition and symmetry), and scale (eight levels). We performed repeated measures ANOVAs to analyze the data.

## 3.2.1. Reaction times

The analysis of RTs was restricted to correct responses. The participants gave significantly faster responses for stimuli presented at longer presentation times (80 ms) than for stimuli presented at shorter presentation times (50 ms), [F(1,11) = 11.92, p < 0.05]. The RTs for symmetrical patterns were significantly shorter than for repeated patterns [F(1,11) = 70.48, p < 0.001]. The main effect of scale did not reach significance for repetition [F(7,5) = 0.84, p = 0.61] nor for symmetry [F(7,5) = 2.2, p = 0.21]. None of the interactions was significant.

## 3.2.2. Discriminability (d')

The results for d' data are shown in Fig. 4. There was no significant main effect of presentation time nor were there significant interactions with presentation time. The d' values for symmetry were significantly higher than for repetition [F(1,11) = 223.46, p < 0.001], the main effect of scale was significant both for repetition [F(7,5) =17.21, p < 0.005] and for symmetry [F(7,5) = 63.09, p < 0.001]. The regularity × scale interaction was also significant [F(7,5) = 7.06, p < 0.05].



Fig. 4. Discriminability measured as d' as a function of the spatial scale in Experiment 1.

### 3.3. Discussion

Both the RT and d' data exhibit the well-known phenomenon that symmetry is generally better detectable than repetition. As mentioned, however, the main objective of this experiment was to investigate number effects in symmetry and, in particular, in repetition.

For symmetry, the d' values remained at a fairly constant level of around 4.5, across scales 3–8. A level of d' = 4.5 is very high, so that this constancy may be a ceiling effect. This questions the earlier-mentioned studies that reported minor or no number effects in symmetry. Generally, these reports were not based on d', but on RTs or accuracy rates. Hence, although the now found constancy of d' does not contradict the holographic prediction, it would be better to test the absence of a number effect at lower d' levels—for instance, in perturbed symmetry (a recent study suggests that, even then, there is no number effect; see van der Helm & Leeuwenberg, 2003).

A special word may be devoted to the unexpected lower d' values for symmetry at the finest scales 1 and 2. These lower d' values were not caused by a low correct hit rate (that was actually very constant across all scales), but by a high false alarm rate. We think that this high false alarm rate was due to the separation bar that we introduced for the reason we mentioned in the stimulus section of the experimental design. That is, using the same stimulus type in an even broader range of fine scales, but without using a separation bar, Oomes (1998) did not find such a high false alarm rate. It seems that the separation bar triggered a more "liberal" matching of corresponding points, which, especially at fine scales, would yield more false alarms. In other words, without the separation bar, we probably would not have found these lower d' values at the finest scales.

The novel result of this experiment is the significant main effect of scale on d' for repetition. This finding is consistent with the holographic approach which, as we

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explicated earlier, predicts computationally a number effect in repetition. As we discuss next, the algorithmic translation of this number effect predicts salience effects as well.

# 4. Salience effects

It is often thought that the detectability of symmetry and repetition improves when the pattern contains salient subpatterns. Indeed, if subpatterns are salient because they exhibit additional regularity, then they seem to improve detectability, with a larger benefit for repetition than for symmetry (Corballis & Roldan, 1974; for an overview and a holographic explanation at the computational level of description, see van der Helm & Leeuwenberg, 1996).

In the second experiment, however, we considered salient substructures that do not exhibit extra local regularities. We introduced relatively small subpatterns (one in each pattern half), that consisted of blobs, i.e., of larger patches than in the rest of the pattern (see Fig. 5). That is, these novel stimuli exhibit only one global regularity (symmetry or repetition), but they differ from the usually investigated cases because they contain such coarse blob areas. We expect that such blob areas are salient because, compared to the rest of the pattern, such blob areas contain predominantly low spatial frequency information which, as Julesz and Chang (1979) put it, seems to have more perceptual weight than high spatial frequency information. Indeed, there is evidence (for an overview, see Palmer, 1999) that low spatial frequency information follows the magnocellular pathway, which is faster than the parvocellular pathway that carries high spatial frequency information.

Furthermore, Tyler and Baseler (1998) found evidence that regularity detection takes place in the middle occipital gyrus (MOG), i.e., after the magno-parvo separation has occurred in the lateral geniculate nucleus. This implies, for our stimuli, a split regularity-detection process in the MOG. That is, the regularity in the blob areas starts to be processed first and, only some time later, the regularity in the rest of the pattern starts to be processed. The question now is: Does such a split pro-



Fig. 5. Examples of the test stimuli used in Experiment 2. (A) A symmetrical pattern and (B) a repetition pattern.

cess improve the detectability of symmetry and repetition, or not?

The holographic approach answers this question by way of the so-called holographic bootstrap model at the algorithmic level of description. This model is a modification of Wagemans et al.'s (1993) original bootstrap model, and we refer the reader to van der Helm and Leeuwenberg (1999) for details about how this modification follows from the computational description within the holographic approach. Here, we sketch its basic idea, and its implication regarding our present stimuli.

Just as the original bootstrap model, the holographic bootstrap model takes correlation quadrangles (trapezoids and parallelograms) as the anchors for the detection of symmetry and repetition (see Fig. 1B). First, suppose that a correlation trapezoid has been found as an indication of the presence of symmetry. Then, just as in the original bootstrap model, the holographic bootstrapping model propagates in steps, each step adding more and more correlation trapezoids in parallel (see Fig. 6A). Thus, the number of correlation trapezoids increases exponentially.

Second, suppose that a correlation parallelogram has been found as an indication of the presence of repetition. Then, the original bootstrapping model propagates essentially the same way as it does for symmetry. In the holographic bootstrapping model, however, the four elements constituting the parallelogram are first clustered into two single units, i.e., into two identical blocks (see Fig. 6B). These two blocks then form one virtual line, for which a new virtual line is sought to form anew a correlation parallelogram. If found, the new elements are included to form two larger identical blocks, and so on until, in accordance with the holographic structure of repetition, the entire pattern has been established as consisting of two identical blocks. Thus, this time the propagation spreads linearly.



Fig. 6. Holographic bootstrapping in a symmetry (A) and in a repetition (B). The bold, dashed, lines indicate correlation quadrangles. The thin, dashed, lines indicate the search for additional correlation quadrangles. The shaded areas in B indicate the stepwise clustering into two larger and larger identical blocks.

Hence, the foregoing algorithmic description implies a linear spreading in repetition, but an exponential spreading in symmetry. This agrees well with the earliergiven computational description of a number effect in repetition but not in symmetry. For instance, in symmetry, the propagation spreads over 35 elements just as fast as it does over 60 elements, so that there is hardly a number effect. The difference in spreading speed has the following remarkable implication for our present stimuli.

Suppose, first, that the two blob areas (one in each half of a global symmetry or repetition) are not processed separately from the rest. That is, the pattern is processed as usual, starting from a correlation quadrangle that happens to relate two points inside one blob area to two points inside the other blob area. Assume that this would take a total processing time of T(pattern). Now suppose that, these blob areas are processed first—taking a time of T(blobs). Subsequently, the rest of the pattern is processed starting from the correlations resulting from these blob areas-taking a time of T(rest). Then, in repetition, the total time T(blobs) + T(rest) is about equal to T(pattern), but in symmetry it is larger than T(pattern). That is, in repetition, the split process clusters the blob areas little by little (i.e., linearly) into two identical single units, just as would be the case without this split process. These two units then form a virtual line from which the rest of the pattern can be processed, as if these units were two basic elements in the pattern. In symmetry, however, by the time the split process has processed the blob areas, the no-split process would already have processed a much larger region, because of the exponential spreading.

For our stimuli, the foregoing suggests that symmetry is hindered by the magno-parvo split process, whereas repetition is not. An additional factor is that the increase of patch size in the blob areas implies a lower number of elements in the blob areas. Because the holographic approach predicts a number effect in repetition but not in symmetry, the holographic prediction regarding the present stimuli is that the salient blob areas have a strengthening effect on repetition but a weakening effect on symmetry. This prediction was tested in the experiment reported next.

# 5. Experiment 2

# 5.1. Method

## 5.1.1. Subjects

The subjects were 18 undergraduate students and staff members (seven males and 11 females) including two authors (GvdV and PvdH) from the University of Nijmegen who had not participated in Experiment 1. Students were either paid or received course credits. The subjects were aged between 18 and 55 and had normal or corrected-to-normal acuity.

#### 5.1.2. Stimuli

In this experiment, two main types of stimuli were produced: symmetry and repetition. The stimuli, black and white Gaussian blob patterns, were generated in the same fashion as in Experiment 1 except for the following. We now introduced two, relatively small, elliptical areas (one in each pattern half) with a patch size that differed from the patch size in the rest of the pattern. Thus, two different coarseness levels are present simultaneously (see Fig. 5). Each stimulus was  $219 \times 250$ pixels and subtended  $4.1^{\circ} \times 4.7^{\circ}$  in the visual field. The elliptical areas were positioned at the center of both stimulus halves. Each elliptical area had a maximal diameter of 100 pixels and a minimal diameter of 50 pixels, and subtended  $1.9^{\circ} \times 0.9^{\circ}$  in the visual field. Furthermore, as illustrated in Figs. 7 and 8, the coarseness difference between the elliptical areas and the larger area was varied. At a starting scale (scale 0), both areas had the same coarseness level (the Gaussian noise was blurred with a Gaussian filter radius of four pixels). From this starting scale, the coarseness level of either the elliptical areas (see Fig. 7a and b) or the larger area (see Fig. 8a and b) was increased. At scale 1 a blurring radius of eight pixels was used to produce the coarser areas, and at scale 2 a blurring radius of 12 pixels.

Within each of the two main conditions (symmetry and repetition), three subconditions (see Figs. 7 and 8) were constructed as follows. Perfect subcondition: the pattern shows symmetry or repetition in the elliptical areas as well as in the larger area. Imperfect subcondition 1: the elliptical areas are random while the larger area shows symmetry or repetition. Imperfect subcondition 2: the elliptical areas show symmetry or repetition while the larger area is random.

Five different stimuli were produced for each scale in the two imperfect subconditions, and 10 different stimuli for each scale in the perfect condition. Each stimulus was presented three times. Therefore one experimental block (symmetry or repetition) included 300 stimuli: [5 scales  $\times$  (10 imperfect patterns + 10 perfect patterns)]  $\times$  3.

## 5.1.3. Apparatus and procedure

The apparatus and the procedure were generally identical to those in Experiment 1 except that, now, the stimulus presentation time was longer (200 ms) and a mask was not presented after the stimuli. Pilot experiments indicated that subjects required these changes to produce accuracy rates similar to those in Experiment 1.

Subjects were instructed to discriminate between patterns that are perfect (perfect symmetry and perfect repetition) and patterns that are imperfect (imperfect symmetry and imperfect repetition). They were told



Fig. 7. (a) Examples of the symmetrical stimuli from each subcondition and scales for the elliptical area in Experiment 2. (A) Perfect subcondition: the patterns show symmetry in the elliptical areas as well as in the larger area. (B) Imperfect subcondition 1: the elliptical areas are random while the larger area shows symmetry. (C) Imperfect subcondition 2: the elliptical areas show symmetry while the larger area is random. (b) Examples of the repetition stimuli from each subcondition and scales for the elliptical area in Experiment 2. (A) Perfect subcondition: the patterns show repetition in the elliptical areas as well as in the larger area. (B) Imperfect subcondition 1: the elliptical areas are random while the larger area shows repetition. (C) Imperfect subcondition 2: the elliptical areas show repetition while the larger area is random.

that, in the imperfect stimuli, only part of the pattern exhibited the relevant regularity.

# 5.2. Results

Unlike the conditions in Experiment 1, the subconditions in this experiment cannot be considered as control conditions of each other, so that, now, d' is not a



Fig. 8. (a) Examples of the symmetrical stimuli from each subcondition and scales for the larger area in Experiment 2. (A) Perfect subcondition: the patterns show symmetry in the elliptical areas as well as in the larger area. (B) Imperfect subcondition 1: the elliptical areas are random while the larger area shows symmetry. (C) Imperfect subcondition 2: the elliptical areas show symmetry while the larger area is random. (b) Examples of the repetition stimuli from each subcondition and scales for the larger area in Experiment 2. (A) Perfect subcondition: the patterns show repetition in the elliptical areas as well as in the larger area. (B) Imperfect subcondition 1: the elliptical areas are random while the larger area shows repetition. (C) Imperfect subcondition 2: the elliptical areas show repetition while the larger area is random.

meaningful criterion to investigate the effect of salient blob areas on symmetry and repetition detection. Instead, for each subcondition separately, we analyzed the accuracy and the **R**T data of the correct responses.

The means and standard deviations of the accuracy and RT data for each subcondition are presented in Tables 1 and 2. In separate figures, the accuracy rates and the mean RTs in the symmetry and repetition conditions are shown as a function of the spatial scale.

Table 1
Means and standard deviations of correct responses in each subcondition

Subconditions	Spatial s	cale										
			For the	elliptical area	ı		For the	larger area				
	0		1		2		1		2			
	М	SD	М	SD	М	SD	М	SD	М	SD		
Perfect												
Symmetry	0.92	0.08	0.81	0.13	0.82	0.16	0.88	0.12	0.89	0.1		
Repetition	0.59	0.19	0.83	0.11	0.9	0.12	0.58	0.13	0.72	0.14		
Imperfect												
Symmetry	0.11	0.17	0.59	0.27	0.75	0.22	0.17	0.15	0.19	0.18		
Repetition	0.55	0.23	0.75	0.21	0.76	0.19	0.43	0.19	0.31	0.19		
Imperfect 2												
Symmetry	0.94	0.09	0.81	0.2	0.67	0.23	0.98	0.03	0.97	0.04		
Repetition	0.64	0.2	0.34	0.19	0.27	0.18	0.67	0.15	0.66	0.15		

Note: *n* = 18.

Table 2Means and standard deviations of RT for correct responses in each subcondition

Subconditions	Spat	ial scale													
				For	the elliptic	cal area				For	the larger	area			
	0			1			2			1			2		
	n	М	SD	n	М	SD	n	М	SD	n	М	SD	n	М	SD
Perfect															
Symmetry	18	507.88	135.42	18	561.69	149.25	18	580.76	157.80	18	528.9	162.73	18	560.9	173.05
Repetition	18	721.90	270.51	18	620.62	218.66	18	592.04	203.45	18	747.05	233.99	18	699.94	200.21
Imperfect 1															
Symmetry	10	640.40	370.15	18	654.96	184.08	18	588.74	144.93	15	877.35	367.14	14	852.92	394.69
Repetition	18	773.56	316.79	18	684.15	190.72	18	645.81	161.99	18	777.67	252.11	17	775.22	268.67
Imperfect 2															
Symmetry	18	542.61	179.52	18	567.52	132.46	18	578.15	173.72	18	473.88	100.21	18	497.04	99.55
Repetition	18	737.05	253.79	18	802.93	288.43	17	776.54	265.55	18	751.63	268.26	18	754.52	252.93

At the starting scale (scale 0) both the larger area and the elliptical area had the same coarseness level. The Fig. 9a and b show the results for increasing the coarseness in only the elliptical areas. The Fig. 10A and B show the results for increasing the coarseness in the larger area only. Repeated measures ANOVAs were performed to analyze the effect of spatial scale in each part of the patterns. The results of these ANOVAs are shown in Tables 3 and 4, and are summarized next.

# 5.2.1. Results for scaling up the elliptical area

The RT data show no significant effects in both imperfect subconditions but, in the perfect subcondition, the main effects of scale for symmetry and repetition, and the regularity  $\times$  spatial scale interaction, are significant.

The accuracy data show significant main effects of scale for symmetry and repetition in all three subconditions, while the regularity  $\times$  spatial scale interaction is

significant in the perfect subcondition and the imperfect subcondition 1. For both symmetry and repetition, the accuracy increases with scale in the imperfect subcondition 1, and decreases in the imperfect subcondition 2. In the perfect subcondition, however, the accuracy increases for repetition but decreases for symmetry.

## 5.2.2. Results for scaling up the larger area

The RT data, for repetition, show no significant main effect of scale in any of the subconditions and, for symmetry, a significant main effect in only the imperfect subcondition 2. The regularity  $\times$  spatial scale interaction is significant in the perfect subcondition and in imperfect subcondition 2.

The accuracy data show, for symmetry, no significant main effect of scale in any of the subconditions and, for repetition, significant main effects in the perfect subcondition (accuracy increases from scale 0 to scale 2) and in the imperfect subcondition 1 (accuracy decreases



Fig. 9. (a) Mean accuracy for scaling up the elliptical area in each subcondition in Experiment 2. (b) RT of correct responses for scaling up the elliptical area in each subcondition in Experiment 2.

from scale 0 to scale 2). The regularity  $\times$  spatial scale interaction is significant in the perfect subcondition and in the imperfect subcondition 1.

# 5.3. Discussion

The results for scaling up the larger area showed significant effects on accuracy for repetition in the per-

fect subcondition and in the imperfect subcondition 1. These findings are in fine agreement with the number effect found in Experiment 1. For symmetry, fine-scaled subpatterns in coarse-scaled surroundings, however, hardly affected subjects' performance. This is consistent with Dakin and Herbert's (1998) finding that the socalled integration region around the symmetry axis is small for high spatial frequencies. The integration region



Fig. 10. (A) Mean accuracy for scaling up the larger area in each subcondition in Experiment 2 and (B) RT of correct responses for scaling up the larger area in each subcondition in Experiment 2.

is the area outside of which stimulus elements do not seem to be processed in symmetry detection. That is, in our stimuli, the fine-scaled subpatterns probably fall outside this integration region.

The results for scaling up the elliptical area show that coarse-scaled subpatterns (or blob areas) in fine-scaled surroundings affect both symmetry and repetition. In the imperfect subcondition 1, where random blob areas correctly signal imperfect regularities, accuracy indeed increases as blob size increases. In the imperfect subcondition 2, where symmetrical or repeated blob areas incorrectly signal perfect regularities, accuracy indeed decreases as blob size increases. Thus, in both imperfect subconditions, the blob areas become more and more decisive, which confirms their salience. Therefore, in the perfect subcondition, where both blob areas and

Table 3
Results of repeated measures of ANOVA of correct responses in each subcondition

Subconditions	Main effec	t of scale			Condition (symmetry, repetition) $\times$ Scale interaction						
	Scale				ScaleFor the elliptical areaFor the larger areaFpFp						
	For the ell	liptical area	For the la	rger area	For the el	liptical area	For the larger area				
	F	р	F	р	F	р	F	р			
Perfect											
Symmetry	4.27	0.03	0.78	0.48	30.03	<0.001	8.07	0.004			
Repetition	18.65	<0.001	10.73	0.001							
Imperfect 1											
Symmetry	47.06	<0.001	1.21	0.33	19.13	<0.001	19.15	<0.001			
Repetition	8.02	0.004	6.72	0.008							
Imperfect 2											
Symmetry	12.39	0.001	2.03	0.16	2.77	0.09	0.05	0.95			
Repetition	14.75	<0.001	0.17	0.85							

Note: df = 16.

Table 4 Results of repeated measures of ANOVA for RT of correct responses in each subcondition

Subconditions	Main	effect of so	cale				Condition (symmetry, repetition) × Scale interaction   Scale   For the elliptical area   df F   df F						
	Scale												
	For the elliptical area			For t	For the larger area			or the elliptical area For the larger area					
	df	F	р	df	F	р	df	F	р	df	F	р	
Perfect													
Symmetry	16	4.79	0.02	16	3.43	0.058	16	15.93	<0.001	16	5.79	0.013	
Repetition	16	5.46	0.02	16	3.56	0.053							
Imperfect 1													
Symmetry	8	2.82	0.12	6	1.12	0.39	8	0.24	0.80	5	1.19	0.38	
Repetition	16	3.29	0.06	15	0.08	0.92							
Imperfect 2													
Symmetry	16	0.52	0.61	16	6.39	0.009	15	0.30	0.74	16	4.33	0.031	
Repetition	15	0.84	0.45	16	0.14	0.87							

surroundings signal perfect regularities, one might expect that the symmetrical or repetitive blob areas would improve performance, and this is indeed the case for repetition but, remarkably, for symmetry, both accuracy and reaction time data deteriorate. This result is perhaps counter-intuitive, but is well in line with the holographic approach which, as we explicated earlier, predicts that such salient blob areas strengthen repetition but weaken symmetry.

One might argue alternatively that the weakening effect in symmetry is due to the spatial separation between the blob areas. After all, Corballis and Roldan (1974) found that a spatial separation between stimulus halves also weakens symmetry. However, Tyler and Hardage (1996) found that symmetry remains about equally detectable when the symmetry halves are simultaneously separated and scaled up. The latter finding is consistent with Dakin and Herbert's (1998) aforementioned finding on integration regions. This suggests, regarding our Experiment 2, that the weakening effect in symmetry cannot be attributed to the spatial separation between the blob areas. Yet, Corballis and Roldan's (1974), Tyler and Hardage's (1996), as well as Dakin and Herbert's (1998) findings concern manipulations of whole symmetry halves and not, as in our case, of parts of symmetry halves. Therefore, in the next experiment we further investigated the modulating effect of integration regions on salience effects.

## 6. Experiment 3

# 6.1. Method

## 6.1.1. Subjects

Sixteen new subjects (six males and 10 females) participated in the experiment. They were aged between 23 and 30 and had (corrected-to-) normal visual acuity. All subjects were again undergraduate or postgraduate students at the University of Nijmegen and were paid or received course credits.

## 6.1.2. Stimuli

The experimental stimuli, black and white Gaussian blob patterns, were generated analogously to the stimuli in the Experiment 2 except for the following. Now, one elliptical area on the center of the symmetry axis was introduced with a size of  $70 \times 110$  pixels  $(1.3^{\circ} \times 2.1^{\circ}$  visual angle). No separation bar was introduced between the two halves of the stimuli.

The coarseness of the elliptical area and the larger area could both be fine (radius of four pixels) or coarse (radius of eight pixels) yielding four combinations: finescaled larger area with fine-scaled elliptical area (FF), coarse-scaled larger area with fine-scaled elliptical area (CF), fine-scaled larger area with coarse-scaled elliptical area (FC) and as a new combination compared to Experiment 2, coarse-scaled larger area with coarse-scaled elliptical area (CC).

Like in Experiment 2, three basic conditions were constructed as follows. Perfect condition: the pattern shows symmetry in the elliptical area as well as in the larger area. Imperfect condition 1: the elliptical area is random while the larger area shows symmetry. Imperfect condition 2: the elliptical area shows symmetry while the larger area is random. Fig. 11 shows example stimuli for the perfect condition.

Twenty different stimuli were produced for each of the four scale combinations in the perfect condition. The two imperfect conditions each consisted of 10 different stimuli for each scale combination. The total experiment consisted of 480 trials: [20 perfect + 10 imperfect 1 + 10



Fig. 11. Examples of the stimuli from each scale of the perfect condition in Experiment 3.

imperfect  $20] \times 2$  background scales  $\times 2$  scales of the elliptical area.

## 6.1.3. Apparatus and procedure

The apparatus and procedure were identical to those in Experiment 2.

# 6.2. Results

We performed separate repeated measures ANOVAs for the three basic conditions (perfect, imperfect 1 and imperfect 2). The coarseness of the background and the coarseness of the elliptical area were within subject variables. We analysed the RT and accuracy as dependent variables. RTs were analysed for correct responses. The results for each condition are shown in Fig. 12. The means and standard deviations of accuracy rates and RTs are presented in Table 5.

## 6.2.1. Results for the perfect condition

The main effect of coarseness for the larger area was significant for both RTs [F(1, 15) = 9.265, p < 0.01] and accuracy rates [F(1, 15) = 10.709, p < 0.01]. Subjects responded slower and accuracy rates were lower in case of a coarse larger area than in case of a fine larger area. Furthermore, the interaction between the coarseness of the larger area and the coarseness of the elliptical area was significant for both RTs [F(1, 15) = 11.76, p < 0.01] and for accuracy [F(1, 15) = 5.58, p < 0.05].

Further investigation of all possible contrasts for this interaction revealed that when the elliptical area was fine-scaled subjects reacted faster and were more accurate in case of a fine-scaled larger area (FF) than in case of a coarse-scaled larger area (CF), [RT: F(1,15) = 28.44, p < 0.001; Accuracy: F(1,15) = 12.55, p < 0.05]. None of the other contrasts reached significance.

# 6.2.2. Results for the imperfect condition 1

The main effect of coarseness for the elliptical area was significant for both reaction times [F(1, 15) = 49.61, p < 0.001] and accuracy rates [F(1, 15) = 28.46, p < 0.001]. Subjects reacted faster and had higher accuracy rates in case of a coarse-scaled elliptical area than in case of a fine-scaled elliptical area. Furthermore, the interaction between the coarseness level of the larger area and that of the elliptical area was significant for RTs [F(1, 15) = 8.86, p < 0.01] but not for accuracy rates.

Further investigation of all possible contrasts for this interaction revealed the following significant results. Subjects responded faster to FF than to CC [F(1, 15) = 32.91, p < 0.001]. Subjects responded faster to FC than to FF [F(1, 15) = 44.41, p < 0.001]. Subjects responded faster to FC than to FF [F(1, 15) = 44.41, p < 0.001]. Subjects responded faster to CC than to CF [F(1, 15) = 16.48, p < 0.01]. Subjects responded faster to FC than to FC [F(1, 15) = 34.34, p < 0.001]. Finally, subjects tended to respond faster to FC than to CC [F(1, 15) = 9.12, p = 0.052].



Fig. 12. RT of correct responses and mean accuracy in each condition in Experiment 3. (FF) Fine-scaled larger area with fine-scaled elliptical area. (CF) Coarse-scaled larger area with fine-scaled elliptical area. (FC) Fine-scaled larger area with coarse-scaled elliptical area. (CC) Coarse-scaled larger area with coarse-scaled elliptical area.

Conditions	Combinations of coarseness levels of the larger (L) and the elliptical (E) areas												
	Fine L-Fin	ie E	Coarse L-I	Fine E	Fine L–Co	arse E	Coarse L-Coarse E						
	М	SD	M	SD	M	SD	М	SD					
Perfect													
Accuracy	0.94	0.06	0.88	0.08	0.93	0.07	0.91	0.06					
RT	650.55	100.98	699.46	119.19	685.21	127.64	686.73	112.99					
Imperfect 1													
Accuracy	0.77	0.21	0.73	0.22	0.96	0.04	0.93	0.12					
RT	706.51	106.05	685.72	116.04	592.28	72.71	629.78	94.91					
Imperfect 2													
Accuracy	0.34	0.18	0.85	0.16	0.37	0.21	0.82	0.14					
RT	796.14	145.49	689.44	135.52	775.47	165.81	674.39	124.73					

Table 5

Note: n = 16.

## 6.2.3. Results for the imperfect condition 2

The main effect for coarseness of the larger area was significant for both RTs [F(1, 14) = 28.25, p < 0.001]and accuracy rates [F(1, 14) = 218.84, p < 0.001]. When the larger area was coarse-scaled subjects were faster and more accurate than when the larger area was finescaled. The interaction between the coarseness level of the larger area and that of the elliptical area was not significant.

# 6.3. Discussion

Like in Experiment 2, the results for the two imperfect conditions clearly confirm the salience of the coarsescaled areas. That is, detection of imperfect regularities improved substantially in both RTs and accuracy data when the random information was coarse-scaled in the elliptical area (the imperfect condition 1) or in the larger area (the imperfect condition 2).

The results for the perfect condition show that finescaled symmetry around coarse-scaled symmetry does not affect performance, but that coarse-scaled symmetry around fine-scaled symmetry gives deterioration. This finding seems to stress the relevance of the integration region. Dakin and Herbert (1998) found that the integration region is an elliptical area around the symmetry axis (with the same orientation). Most importantly, they also found that it is larger for coarser scales (without

affecting symmetry detection; see Dakin & Watt, 1994; Oomes, 1998; Tapiovaara, 1990). This seems to explain why fine-scaled symmetry around coarse-scaled symmetry has no effect: The fine-scaled area simply falls outside the integration region for fine scales. It also seems to explain why coarse-scaled symmetry around fine-scaled symmetry may have effect: The fine-scaled area falls at least partly inside the integration region for fine scales, and the coarse-scaled area falls at least partly inside the integration region for coarse scales. It does not explain, however, the deterioration in the latter case.

Hence, if one takes integration regions into account, then Experiment 3 can be said to replicate the pattern of results in Experiment 2. In both cases, symmetry detection is hindered if coarse-scaled symmetry is integrated with fine-scaled symmetry—even though, or perhaps precisely because, the symmetry signals from both areas are about equally strong. This is next discussed further.

# 7. General discussion

Mach (1886/1959) was probably the first to point out that symmetry is better detectable than repetition. Since then, many explanations of this phenomenon have been proposed, but these explanations generally failed when applied to other detectability phenomena concerning perfect and perturbed regularities (for an overview, see van der Helm & Leeuwenberg, 1996, 1999). The most often proposed explanatory factor is proximity: In repetition, the matching process has to bridge a fixed distance between corresponding points, whereas, in symmetry, it can start with corresponding points near the axis of symmetry. Proximity then is not so much conceived as a Gestaltlike grouping principle, but rather as an indication of the range (i.e., the so-called integration region) over which pattern elements may cause neural interactions. In Marr's (1982) terms, proximity would thus be a factor at the implementational level. It would, however, merely be a factor that (co)determines the arena within which the detection process operates. That is, it does not determine what happens within that arena. Now, our experiments were meant to give further understanding of what happens inside that arena.

The main result of Experiment 1 was that we found a gradual increase of d' for repetition as the scale gets coarser. We think that this can be attributed to the fact that lower spatial frequencies are believed to be mediated by fewer but larger receptive fields (cf. Palmer, 1999). That is, in our coarser-scaled repetitions, the fewer but larger blobs are probably processed by way of fewer but larger receptive fields. This suggests implementational compliance with the holographic models at the computational and algorithmic levels, which both predict a number effect in terms of blobs.

Although Experiment 1 did not yield clear-cut results for symmetry, we think that symmetry does not exhibit a number effect. That is, as van der Helm and Leeuwenberg (1999) argued, the symmetry of coarse blob shapes can, logically, be assessed at a fine scale only. This agrees with Dakin and Watt's (1994) empirical finding that symmetry detection matches the performance of a fairly fine-scale filter. Hence, it seems that symmetry, in agreement with the holographic approach, is processed by way of a more or less constant number of receptive fields, no matter the coarseness level.

Furthermore, a qualitative understanding of the results in Experiments 2 and 3 might start from the implementational distinction between the magnocellular and parvocellular pathways, as follows. In our symmetry stimuli, the symmetry signal from the fine-scaled area near the symmetry axis is probably fairly comparable in strength to the symmetry signal from the coarse-scaled blob areas (as follows from Tyler & Hardage's, 1996, finding mentioned earlier). Because these signals occur at different, implementationally separated, spatial frequency levels, these signals might well be difficult to integrate or might even engage in a sort of competition or mutual inhibition (cf. Hughes, Nozawa, & Kitterle, 1996). That is, communication between two different spatial frequency levels is probably much more difficult than communication within one spatial frequency level. In our repetition stimuli, such a competition or inhibition is less likely to occur, because the repetition signal from the coarse-scaled blob areas probably strongly dominates the repetition signal from the fine-scaled surrounding areas.

The foregoing description sounds plausible but is also, as said, qualitative. That is, competition, inhibition, and difficult integration or communication, are terms that describe process effects, rather than the process itself. The holographic approach complements this qualitative description with more specific details. As explicated earlier, the holographic approach implies, computationally, a number effect in repetition but not in symmetry, which can be translated faithfully to the algorithmic level, implying a linear process for repetition but an exponential process for symmetry. As explicated earlier as well, given the magno-parvo distinction this holographic processing difference between repetition and symmetry implies the same effects as implied by the foregoing qualitative description. In other words, the holographic approach complements the implementational magno-parvo distinction with algorithmic details about the process itself-algorithmic details that, in turn, were derived from computational considerations.

## 8. Summary and conclusion

In this paper, we reported on three experiments in which we investigated differential effects of spatial scaling on symmetry and repetition perception. The three experiments were triggered by the holographic approach which predicts, among other things, that repetition detection is affected by varying the number of elements in a pattern, whereas symmetry detection is not. The data of the first experiment showed that this is indeed the case for black and white patch patterns in which we increased patch size in the entire pattern, yielding fewer but larger patches (or blobs). In the second and the third experiment, we increased patch size in small subpatterns only, yielding salient blob areas. In these cases, the data confirmed a further prediction by the holographic approach, i.e., the prediction that salient subpatterns facilitate repetition detection but hinder symmetry detection.

Both holographic predictions regarding differences between symmetry and repetition were based on the difference in holographic structure between repetition and symmetry: Holographically, repetition gets a block structure and symmetry a point structure. This structural difference is compatible with repetition detection propagating linearly and symmetry detection propagating exponentially. These structural and processing differences explain not only the well-known phenomenon that symmetry is better detectable than repetition, but also the now found number effects and salience effects. Regarding the now found salience effects, the holographic explanation complements implementational ideas about the processing of spatial frequencies.

In sum, the holographic approach provides a good starting point for explaining the empirical results, in a way that runs compatibly from the computational, via the algorithmic, to the implementational level of description. That is, this article shows that more detailed knowledge about the structures to be detected may lead the way to more detailed knowledge about the detection process itself.

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